Surname	Centre Number	Candidate Number
First name(s)		2

GCE AS/A LEVEL

S24-2300U10-1

2300U10-1

WEDNESDAY, 15 MAY 2024 – MORNING

MATHEMATICS – AS unit 1
PURE MATHEMATICS A

2 hours 30 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	4	
2	3	
3	3	
4	3	
5	4	
6	7	
7	11	
8	4	
9	9	
10	6	
11	4	
12	10	
13	8	
14	8	
15	7	
16	10	
17	7	
18	12	
Total	120	

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. Given that $y = 12\sqrt{x} - \frac{27}{x} + 4$, find the value of $\frac{dy}{dx}$ when $x = 9$. [4]

2. Find all values of θ in the range $0° < \theta < 180°$ that satisfy the equation

$$2\sin 2\theta = 1 .$$ [3]

Examiner only

3. Find $\int\left(5x^{\frac{1}{4}}+3x^{-2}-2\right)\mathrm{d}x$. [3]

4. Given that n is an integer such that $1 \leqslant n \leqslant 6$, use proof by exhaustion to show that n^2-2 is not divisible by 3. [3]

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5. A triangle *ABC* has sides *AB* = 6 cm, *BC* = 11 cm and *AC* = 13 cm. Calculate the area of the triangle. [4]

Examiner only

6. (a) Find the exact value of x that satisfies the equation

$$\frac{7x^{\frac{5}{4}}}{x^{\frac{1}{2}}} = \sqrt{147}\,.$$ [4]

(b) Show that $\dfrac{(8x-18)}{(2\sqrt{x}-3)}$, where $x \neq \dfrac{9}{4}$, may be written as $2(2\sqrt{x}+3)$. [3]

Examiner only

7. (a) The line L_1 passes through the points $A(-3, 0)$ and $B(1, 4)$. Determine the equation of L_1. [3]

(b) The line L_2 has equation $y = 3x - 3$.

(i) Given that L_1 and L_2 intersect at the point C, find the coordinates of C.

(ii) The line L_2 crosses the x-axis at the point D. Show that the coordinates of D are (1, 0). [4]

Examiner only

(c) Calculate the area of triangle *ACD*. [2]

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(d) Determine the angle *ACD*. [2]

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8. Prove that $x - 10 < x^2 - 5x$ for all real values of x. [4]

Examiner only

9. (a) Write down the binomial expansion of $(2-x)^6$ up to and including the term in x^2. [3]

Examiner only

(b) Given that

$$(1+ax)(2-x)^6 \equiv 64+bx+336x^2+\ldots\ ,$$

find the values of the constants a, b. [6]

Examiner only

10. Water is being emptied out of a sink. The depth of water, y cm, at time t seconds, may be modelled by

$$y = t^2 - 14t + 49 \qquad 0 \leqslant t \leqslant 7.$$

(a) Find the value of t when the depth of water is 25 cm. [3]

(b) Find the rate of decrease of the depth of water when $t = 3$. [3]

Examiner only

11. (a) Sketch the graph of $y = 3^x$. Clearly label the coordinates of the point where the graph crosses the y-axis. [2]

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(b) On the **same set of axes**, sketch the graph of $y = 3^{(x+1)}$, clearly labelling the coordinates of the point where the graph crosses the y-axis. [2]

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12. A curve *C* has equation $y = -x^3 + 12x - 20$.

(a) Find the coordinates of the stationary points of *C* and determine their nature. [7]

Examiner only

(b) Determine the range of values of x for which the curve is decreasing.
Give your answer in set notation. [3]

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Examiner only

13. The position vectors of the points *A* and *B*, relative to a fixed origin *O*, are given by

$$\mathbf{a} = 4\mathbf{i} + 7\mathbf{j}, \qquad \mathbf{b} = \mathbf{i} + 3\mathbf{j},$$

respectively.

(a) Find the vector **AB**. [2]

(b) Determine the distance between the points *A* and *B*. [2]

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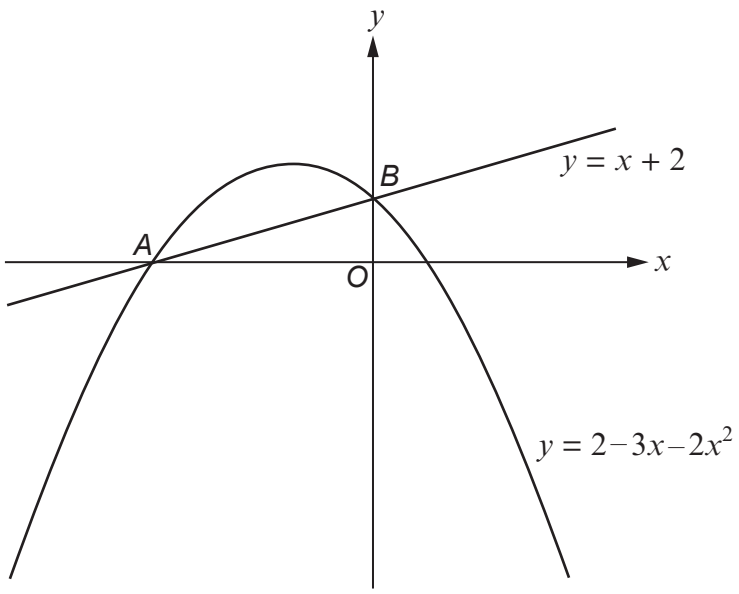
(c) The position vector of the point *C* is given by $\mathbf{c} = -2\mathbf{i} + 5\mathbf{j}$. The point *D* is such that the distance between *C* and *D* is equal to the distance between *A* and *B*, and *CD* is parallel to *AB*. Find the possible position vectors of the point *D*. [4]

Examiner only

14. The diagram below shows a sketch of the curve *C* with equation $y = 2 - 3x - 2x^2$ and the line *L* with equation $y = x + 2$. The curve and the line intersect the coordinate axes at the points *A* and *B*.



(a) Write down the coordinates of *A* and *B*. [2]

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(b) Calculate the area enclosed by C and L. [6]

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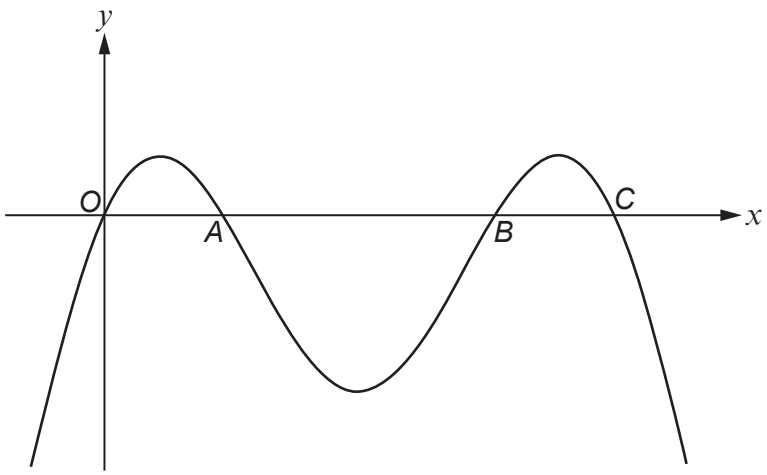
15. The diagram shows a sketch of part of the curve with equation $y = 2\sin x + 3\cos^2 x - 3$.
The curve crosses the x-axis at the points *O*, *A*, *B* and *C*.



Find the value of x at each of the points *A*, *B* and *C*. [7]

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16. (a) Find the range of values of k for which the quadratic equation $x^2 - kx + 4 = 0$ has no real roots. [4]

(b) Determine the coordinates of the points of intersection of the graphs of $y = x^2 - 3x + 4$ and $y = x + 16$. [4]

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(c) Using the information obtained in parts (a) and (b), sketch the graphs of $y = x^2 - 3x + 4$ and $y = x + 16$ on the same set of axes. [2]

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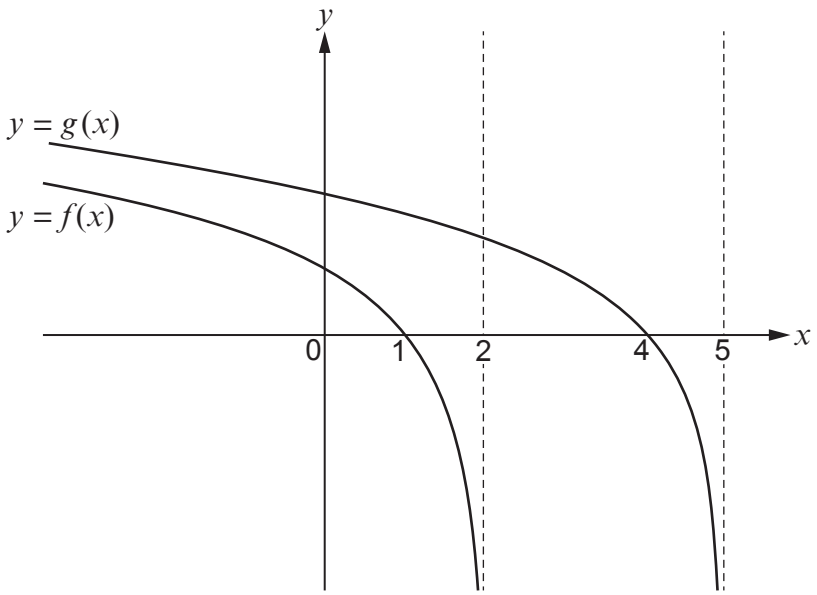
17. A function f is defined by $f(x) = \log_{10}(2-x)$. Another function g is defined by $g(x) = \log_{10}(5-x)$. The diagram below shows a sketch of the graphs of $y = f(x)$ and $y = g(x)$.



(a) The point (c, 1) lies on $y = f(x)$. Find the value of c. [2]

Examiner only

(b) A point *P* lies on $y = f(x)$ and has x-coordinate α. Another point *Q* lies on $y = g(x)$ and also has x-coordinate α. The distance between *P* and *Q* is 1·2 units. Find the value of α, giving your answer correct to three decimal places. [5]

Examiner only

18. (a) A circle *C* has centre (–3, –1) and radius $\sqrt{5}$. Show that the equation of *C* can be written as $x^2 + y^2 + 6x + 2y + 5 = 0$. [2]

(b) (i) Find the equations of the tangents to *C* that pass through the origin *O*. [6]

(ii) Determine the coordinates of the points where the tangents touch the circle. [4]

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END OF PAPER

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

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